Fundamental Properties of Radiation from a Leaky Mode Excited on a Planar Transmission Line

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Abstract — **A normalized leaky-mode radiation field that is applicable to a leaky mode excited by a source on *any* printed-circuit structure is derived. The general properties of the leaky-mode radiation field are examined in detail for a variety of phase and attenuation constants as well as distances from the source. The results provide a general view of leaky-mode radiation properties independent of the type of planar transmission line that supports such a mode.**

I. INTRODUCTION

The existence of dominant (quasi-TEM) leaky modes on printed-circuit transmission lines has been a subject of considerable interest. These modes are usually undesirable and have been found on a number of planar transmission line structures including multilayer stripline structures, microstrip lines, coplanar strips, and CPW [1-4].

Most studies involving leaky modes have focused on a specific structure, showing results for the wavenumber of the leaky mode, or (more recently) for the current or field of the leaky mode when excited by a source [1, 2]. Hence, the conclusions from these studies are tied to specific structures (although many of the conclusions are applicable to a variety of structures).

It is shown here that by using a specific normalization, the fields radiated by a leaky mode that is excited by a source on *any printed-circuit structure* can be represented in the same unified way. In this summary, the calculation of this normalized leaky-mode field is presented. The results apply to any type of printed transmission line (microstrip, coplanar waveguide, multiplayer stripline, coupled lines, etc.) that is excited by a source, provided the leakage occurs only into the TM_0 mode of the background structure. The characteristics of the leaky-mode fields are studied in detail as a function of the leaky-mode phase constant (leakage angle) and leakage constant, although results are presented in this summary for one leakage angle only. The purpose of this work is to draw general conclusions about the leaky-mode fields that are excited by a gap source, independent of the structure. Results are only shown for "physical" leaky modes, although results for leaky modes in the "spectral-gap" region can also be obtained [5].

II. SUMMARY OF ANALYSIS

A diagram of a general planar transmission line structure with a small, but finite 1-V gap source is shown in Fig. 1. Although the figure shows a covered microstrip configuration, the analysis applies to any printed-circuit structure. The conducting strip is assumed to be infinite in the $\pm z$ directions and all of the conductors are assumed to be perfect. For simplicity, the strip width W is assumed to be small so that the transverse component of the current can be neglected.

Assuming that the TM_0 mode is the only mode above cutoff, the fields radiated by the strip current into this mode (i.e., the leakage fields) can be formulated by calculating the fields radiated into this mode by a z-directed infinitesimal electric dipole, and then integrating over the strip current [1],

$$E_y(x,y,z) = A(y) \int_{-\infty}^{+\infty} I(z') H_1^{(2)}\left(k_{TM_0}\rho'\right)\cos(\phi')\,dz', \quad (1)$$

where

$$\rho' = \sqrt{(x)^2 + (z-z')^2}, \quad (2)$$

$$\cos(\phi') = \frac{z-z'}{\rho'}, \quad (3)$$

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and $A(y)$ is a function that gives the y-variation of the fields due to the dipole, which can be calculated using standard spectral-domain techniques.

To obtain a general normalized form for the leakage field, the strip current $I(z')$ is assumed to be a leaky-mode current that is bi-directionally excited by a gap source, all the distances in (1) are normalized by λ_{TM_0} (i.e. $\bar{x} = x/\lambda_{\text{TM}_0}$, $\bar{z} = z/\lambda_{\text{TM}_0}$, $\bar{\rho} = \rho/\lambda_{\text{TM}_0}$), and the phase and attenuation constants are normalized by k_{TM_0} (i.e. $\bar{\beta}_z = \beta_z/k_{\text{TM}_0}$, $\bar{\alpha}_z = \alpha_z/k_{\text{TM}_0}$). After some simple manipulation, the leaky-mode vertical electric field becomes

$$E_y(\bar{x},\bar{y},\bar{z}) = A_N \cdot \int_{-\infty}^{+\infty} e^{-j(\bar{\beta}_z - j\bar{\alpha}_z)2\pi|\bar{z}'|} H_1^{(2)}(2\pi\bar{\rho}')\cos(\phi')d\bar{z}', \quad (4)$$

where

$$A_N = A(y)' A_{LM} \lambda_{TM_0} \quad (5)$$

is a constant (for a fixed value of y) that only depends on the material parameters of the specific structure. In this expression, A_{LM} is the complex excitation coefficient for the leaky mode. Note that, to within a multiplicative factor, the leaky-mode field at a given normalized position ($\bar{x}$, $\bar{z}$) on any printed circuit line, in any layered structure, is solely a function of the normalized variables $\bar{\alpha}_z$ and $\bar{\beta}_z$. For the results presented in this summary, the magnitude of normalized leakage field quantity represented by the integral in (4) will be plotted. The fields for any specific structure may be obtained by scaling these results by the constant A_N in (5).

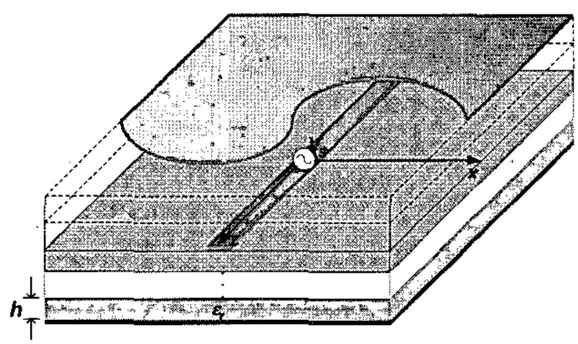

Fig. 1. The geometry of a general planar transmission line with a finite gap source V_s.

For some of the results, the geometrical optics (GO) field is plotted for comparison. The GO field is that predicted by "ray optics" theory, and should be valid in the limit $\alpha_z \to 0$. The magnitude of this field for the leaky-mode structure (with bi-directional excitation) can be written as

$$|\text{GO}(\bar{x},\bar{z})| \propto \begin{cases} e^{-2\pi\bar{\alpha}_z|\bar{z}|}e^{+2\pi\bar{\alpha}_x|\bar{x}|} \; ; (\bar{x},\bar{z}) \text{ in the lit region} \\ 0 \quad ; \text{otherwise} \end{cases} ,(6)$$

where $\bar{\alpha}_x > 0$, $\bar{\alpha}_z > 0$, and the lit region is the angular region between the axis of the strip and the lines drawn from the source that define the leakage angle. In the results presented in this summary, the GO field is normalized so that it agrees with the exact leaky-mode field at the maximum of the exact field.

III. RESULTS

In this summary, the general characteristics of the leaky-mode field are examined using a case where the predicted leakage angle (accurate for small α_z),

$$\phi_0 = \cos^{-1}(\bar{\beta}_z), \quad (7)$$

is specified to be $\phi_0 = 50°$, thereby fixing the value of $\bar{\beta}_z$. The results for three normalized attenuation constants are shown in Figures 2-4 ($\bar{\alpha}_z$ = 0.001, 0.01, and 0.1, respectively).

In Figure 2 ($\bar{\alpha}_z$ = 0.001) the radiation field inside the triangular lit region is strong. This radiation picture closely resembles that for the GO field. Note that the GO field strength is uniform along the lines of leakage within the lit region. The results in Figure 3 demonstrate that when $\bar{\alpha}_z$ is increased to 0.01, the overall field strength is decreased, but the radiation shape becomes more focused along the leakage angle. That is, more of a "beam" exists in the near field. Further increase in $\bar{\alpha}_z$ yields radiation that is diffuse with no well-defined lit region or beam, as shown in Fig. 4, which is characteristic of radiation from a localized current near the gap source.

Figures 2-4 also show some other general properties created by changes in $\bar{\alpha}_z$. The gradient of the field in the shadow region, along a direction perpendicular to the shadow boundary, is large for $\bar{\alpha}_z$ = 0.001, creating an abrupt decay of the field past the shadow boundary. The gradient within the lit region is fairly small (GO predicts that this gradient should become smaller as $\bar{\alpha}_z$ decreases). As $\bar{\alpha}_z$ is increased, this gradient becomes smaller in the shadow region, and larger in the lit region, eventually blurring the definition of the lit region. Also, the field magnitude is changing more rapidly along the leakage angle direction as $\bar{\alpha}_z$ increases.

Polar plots of the normalized leakage field are given in Figs. 5-7. These results illustrate the evolution of the shape of leakage field for $\phi_0 = 50°$ and $\bar{\alpha}_z$ = 0.001, 0.01, and 0.1. For very small $\bar{\alpha}_z$ ($\bar{\alpha}_z$ = 0.001), the shape of the field varies slowly with distance and the field magnitude along the leakage angle continually increases even up to $\bar{\rho} = 100$. For $\bar{\alpha}_z = 0.01$, the shape of the field changes more quickly with distance. The field magnitude along the leakage angle still increases with increasing distance from the source, although at a slower rate. The shape of the field changes still more rapidly with distance when $\bar{\alpha}_z$ is increased to 0.1, and for this larger $\bar{\alpha}_z$ the field strength has switched trends completely along the leakage angle, exhibiting a general decrease with distance. This is because the radiated field changes from a near-field leakage field to a far-field type of pattern at relatively small distances, since the effective length of the radiating strip current is smaller.

Figures 8 and 9 show a comparison between the GO and the exact leaky-mode fields for $\bar{\rho} = 10$ and 100. At $\bar{\rho} = 10$, the agreement is good. The overall shape of the exact leaky-mode field agrees well with the shape of the GO field. The oscillations in the exact leaky-mode field correspond to the interference between a GO leaky-mode field and a source discontinuity radiation. Past the shadow boundary there is only the source discontinuity radiation, so the exact leaky-mode field is smooth and decays monotonically. For $\bar{\rho} = 100$, the agreement is not as good, but the general shape is similar.

These results lead to some general conclusions. For small $\bar{\alpha}_z$, the leaky-mode fields agree well with GO. This agreement becomes better and extends over a larger range of $\bar{\rho}$ as α_z approaches zero. In general, GO does not agree well with the exact leaky-mode field near the source or in the far field. Once $\bar{\alpha}_z$ gets too large the leaky-mode field fails to agree well with GO at any value of $\bar{\rho}$. By comparing the exact and the GO fields it is observed that the oscillations in the lit region are a result of the interference between the leaky-mode field and a source-discontinuity radiation. On the other side of the shadow boundary, the oscillations cease and the field magnitude decreases smoothly since only the source-discontinuity radiation is present in this region.

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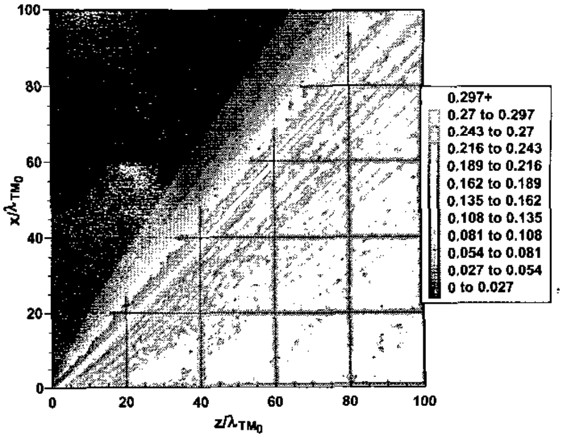

Fig. 2. Vertical component of the normalized leaky-mode electric field for $\bar{\alpha}_z = 0.001$ and $\phi = 50°$.

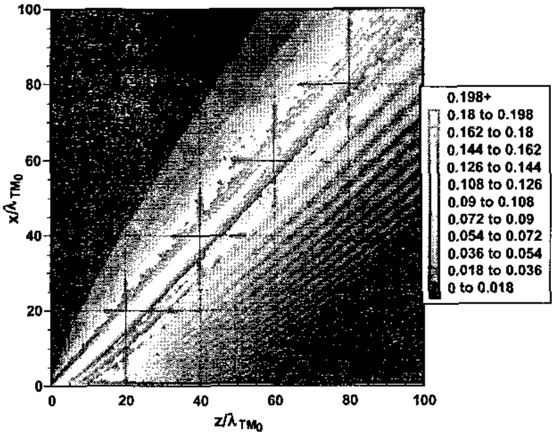

Fig. 3. Vertical component of the normalized leaky-mode electric field for $\bar{\alpha}_z = 0.01$ and $\phi = 50°$.

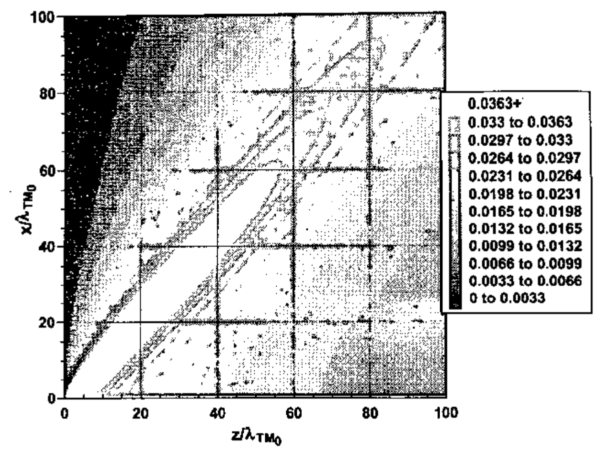

Fig. 4. Vertical component of the normalized leaky-mode electric field for $\bar{\alpha}_z = 0.1$ and $\phi = 50°$.

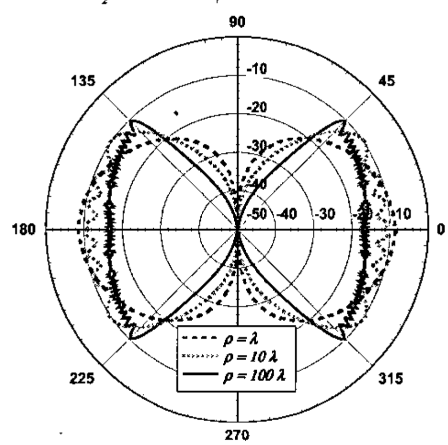

Fig. 5. Vertical component of the normalized leaky-mode electric field (dB) for $\bar{\alpha}_z = 0.001$ and $\phi = 50°$ for $\bar{\rho} = 1, 10, 100$.

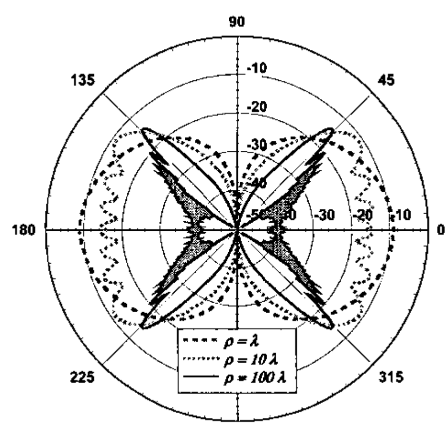

Fig. 6. Vertical component of the normalized leaky-mode electric field (dB) for $\bar{\alpha}_z = 0.01$ and $\phi = 50°$ for $\bar{\rho} = 1, 10, 100$.

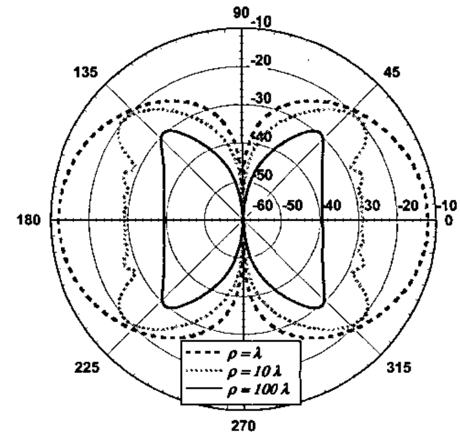

Fig. 7. Vertical component of the normalized leaky-mode electric field (dB) for $\bar{\alpha}_z = 0.1$ and $\phi = 50°$ for $\bar{\rho} = 1, 10, 100$.

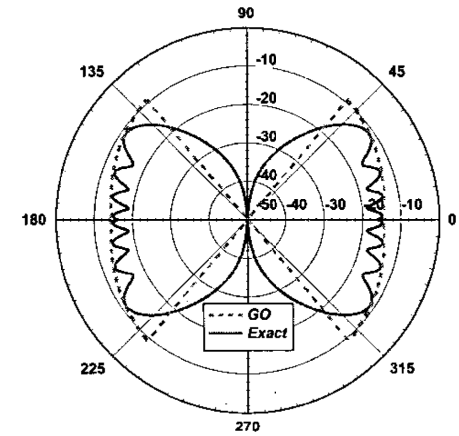

Fig. 8. Vertical component of the normalized leaky-mode electric field (dB) with geometrical optics for $\bar{\alpha}_z = 0.01$, $\phi = 50°$, and $\bar{\rho} = 10$.

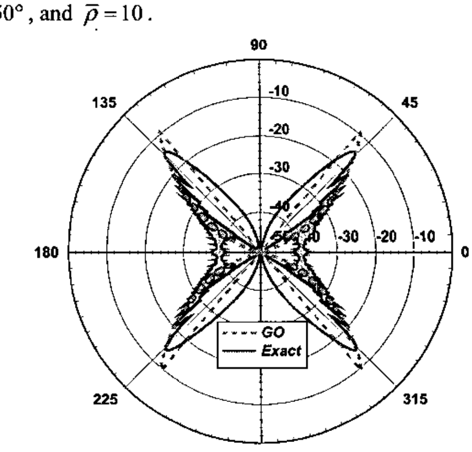

Fig. 9. Vertical component of the normalized leaky-mode electric field (dB) with geometrical optics for $\bar{\alpha}_z = 0.01$, $\phi = 50°$, and $\bar{\rho} = 100$.